Applying Wavelet Analysis Methods to Processing of Electrocardiographical Data

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1. Abstract

Wavelet analysis formalism developing since the beginning of 1980s drastically expands the area of its usage. This paper covers some wavelet based methods for processing of electrocardiographical (ECG) data. We used one type of wavelets in our calculations only but proposed algorithms will still work with other types of them, including wavelets of second generation.

Sections 2–4 briefly describe the basics of wavelet-analysis, further in section 6 we present methods of visualization and handling of ECG signals with wavelets. Procedures of suppression of ECG distortions (such as noise and corruption of iso-lile) and selecting *R–R* intervals are proposed.

Presented in section 7 method of adaptive filtering which enhances the quality of data processing has its own significance and may be fruitfully applied in researches of various types of data.

Automatization of ECG processing is particulary actual when handling with large amounts of data, for example in Holter monitoring.

Section 8 contains a brief description of applying wavelets for classification of risk groups of patients based on analysis of heartbeat intervals.

2. Wavelet transform

The term *wavelet transform* means two types of transformation — direct and inverse, those that convert the function $f(x)$ into the set of wavelet coefficients $W_\psi(a,b)$ and vice versa. There are discrete (DWT) and continuous (CWT) versions of both of them. We restrict ourselves of working with CWT only.

Direct wavelet transforms converts the signal according to the following rule:

$$W_\psi(a,b) = \frac{1}{\sqrt{C_\psi}} \int \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right) f(x)\,dx \ , \tag{1}$$

where a and b are the parameters that define scale and shift of function ψ, called analyzing wavelet, C_ψ is a normalizing constant.

With known set of coefficients $W_\psi(a,b)$ it is possible to reconstruct the initial function $f(x)$:

$$f(x) = \frac{1}{\sqrt{C_\psi}} \int\int \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right) W_\psi(a,b) \frac{da\,db}{a^2}. \tag{2}$$

Direct (1) and inverse (2) transforms depend on some function $\psi(x) \in L^2(\mathbf{R})$, which is called basic wavelet. The almost one restriction for its choice if the requirement of existing of normalizing constant

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega = 2\int_0^{\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty \tag{3}$$

A set of functions satisfying this condition is rather wide, so it is possible to select the most appropriate one for the solving task.

Significant part of wavelet theory is developed by I. Daubechies in [1].

3. Types of wavelets

As we mentioned earlier, members of the wavelet family have to satisfy the admissibility condition (3); one of such a family is so called vanishing momenta wavelet family (VMWF) which consists of Gauss exponent derivatives:

$$g_n(x) = (-1)^{n+1} \frac{d^n}{dx^n} e^{-x^2/2}. \quad (4)$$

Normalizing coefficient is the following:

$$C_{g_n} = 2\pi(n-1)!, \quad 0 < n < \infty. \quad (5)$$

The family got its own name because of zero first $n-1$ moments of the members $g_n(x)$:

$$\int_{-\infty}^{\infty} x^m g_n(x) dx = 0 \quad \forall m, 0 \le m < n, \; n \in \mathbf{Z}. \quad (6)$$

VMWF wavelets of first orders are most popular:

$$g_1(x) = -x e^{-x^2/2}, \quad (7)$$

$$g_2(x) = (1 - x^2) e^{-x^2/2}, \quad (8)$$

$$g_3(x) = (x^3 - 3x) e^{-x^2/2}, \quad (9)$$

$$g_4(x) = (-x^4 + 6x^2 - 3) e^{-x^2/2}. \quad (10)$$

Paper [2] covers in detail some properties of these wavelets.

Although continue wavelet transform needs substantial time and computational resorces it can provide much more descriptive results comparing with discrete versions of wavelet transform [7]. Recently fast CWT algorithms were developed [3].

4. Wavelet spectrum and skeleton

Wavelet transform converts one-dimensional function $f(x)$ into a double-argument function $W_\psi(a,b)$, so we obtain a 2D surface over the plane of parameters a and b as a result. Its visual representation is rather intricate and cannot be used in practice. Commonly the set of wavelet coefficients is depicted as a projection of that surface onto (a,b) plane. The value of W_ψ is reproduced via intensity of the color. For example we can draw higher values as light colors while smaller values are dark.

An image build with such a scheme is called wavelet spectrum. Horizontal axis of it corresponds to shifts b, and vertical one to scales a. If you need to view coefficients in wide ranges it is better to show the vertical axis in logarithmical scale.

Although being very informative wavelet spectrum contains redundunt data. It is often enough to know only the positions of local maxima (and/or minima) at the surface. Location of extremums visualizes so called wavelet skeleton which is built from spectrum via selecting local maxima at each scale and putting dark dots at their positions. As a result of this procedure we obtain an image containing the set of lines, or the wavelet skeleton.

Examples of skeletons are shown further in this paper. Publication [4] includes much more detailed description of spectra and skeletons and their abilities in data processing.

5. Electrocardiogram (ECG)

Recording of electrical activity of the human heart is one of the most known and common way of analyzing heart functioning. Registration is usually means projecting electrical heart vector onto several planes, called leads. We use for simplicity second standard lead further. This means that the positive electrode of an registrator contacts with left foot, and negative one — with right hand. Information on ECG registration and analysis can be found in [5] for example.

Figure 1 shows a sample of three periods of real ECG at the lead II. Some pulses are well distinguished on it.

Pulse p corresponds to contractions of left and right auricles.

QRS complex that includes three pulses reflects the period of ventricles activity. Note that q and s pulses might be weak or absent.

T pulse is a projection of ventricles repolarization vector.

An ECG appearance varies from one lead to another, we use lead II only as it was mentioned before.

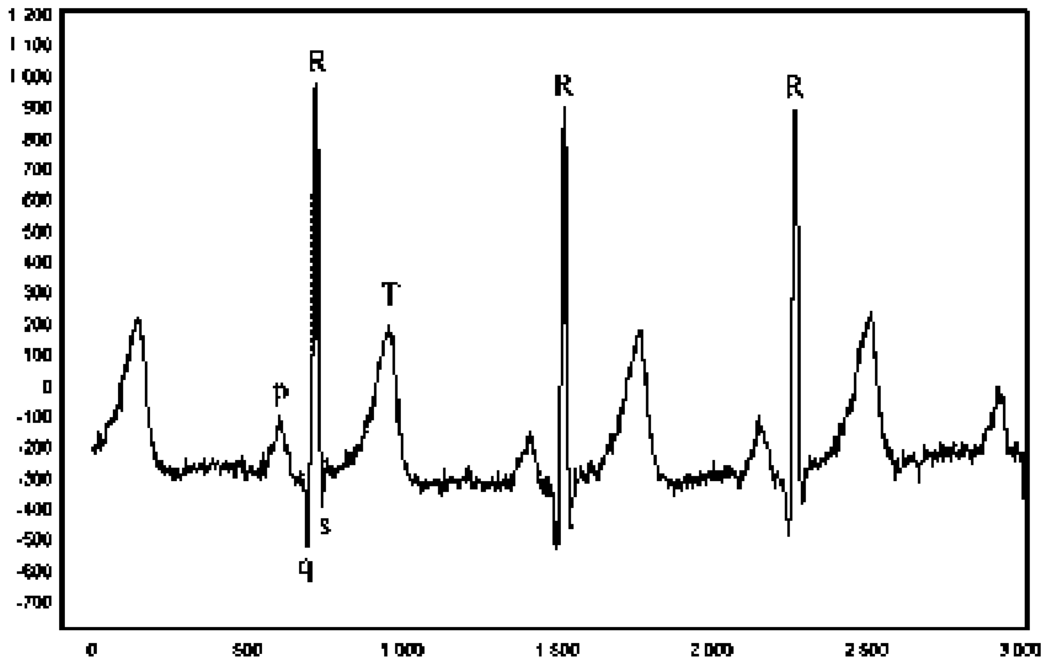

Fig. 1. *Three ECG periods (lead II). An ECG was recorded at the discretization frequency 1 kHz. Numbers on horisontal axis are samples numbers, vertical axis shows the value of the signal (in microvolts).*

One of the main ECG features is *R–R* interval, an interval between two neighbouring R pulses. This parameter reflects the frequency of heartbeats. Extraction of either R pulses or QRS complexes if the first task in automated ECG analysis. It may be significantly hard when we deal with long sequences of ECG periods, for instance, in 24-hour Holter monitoring [6].

Together with an ECG itself the record contains various additive components: power supply 50 Hz background, high frequency noise caused by muscle tremor and low frequency (fractions of Hz) baseline which appears because of chanches of electrical resistans of the sensor–skin contact.

It is possible to exclude noise additions from the signal with the help of low pass filter but at the same time we can obtain widening of sharp pulses inside QRS complexes.

In the next sections we describe the possibilities of wavelet analysis in processing of electrocardiographical data.

6. Analyzing ECG data

Let us view the skeleton of an ECG signal. Fig. 2 shows one made for the signal containing three periods of ECG (see Fig. 1).

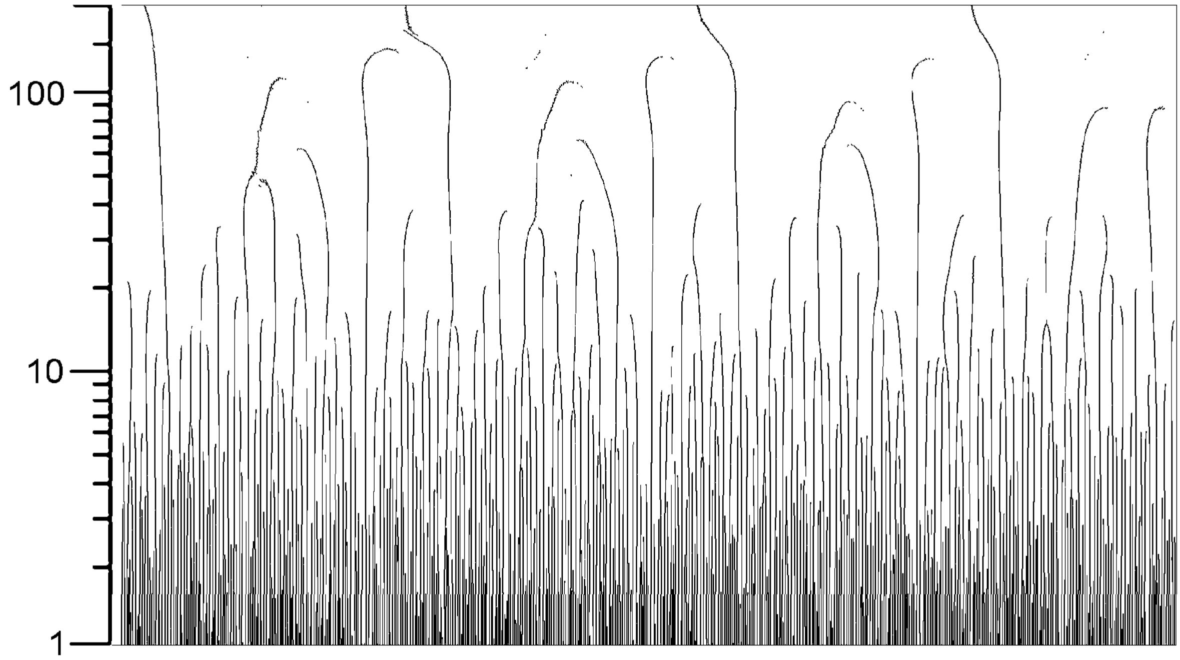

Fig. 2. *Wavelet skeleton of an ECG from fig. 1. Horizontal axis is an axis of shift value b and vertical axis (depicted in logarithmical scale) corresponds to scales a.*

The skeleton clearly shows the structure of the singnal under consideration. Likewise Fourier transform it mirrors the distribution of different frequencies in the signal. Wavelet analysis gives us one more degree of freedom showing *locations* of different components. Note close lines in the bottom part of the skeleton at Fig. 2. These traces arise from high frequency noise in ECG record. Top part an the other hand includes only lines caused by low frequensy componets such as QRS complexes.

6.1. Extracting features

Amalyzing skeleton closer we can select several stripes on it (see Fig. 3). In other words, we split the signal into different areas in frequency space as it was in the case of Fourier transform with the rectangle window. Inverse scale a^{-1} in the wavelet analysis ia an analog of Fouriear frequency ω.

Each stripe on the spectrum selects components of the signal whose frequencies lies within vertical bounds of the stripe. For example, gray one on Fig. 3 includes an area of scales 20 trough 50.

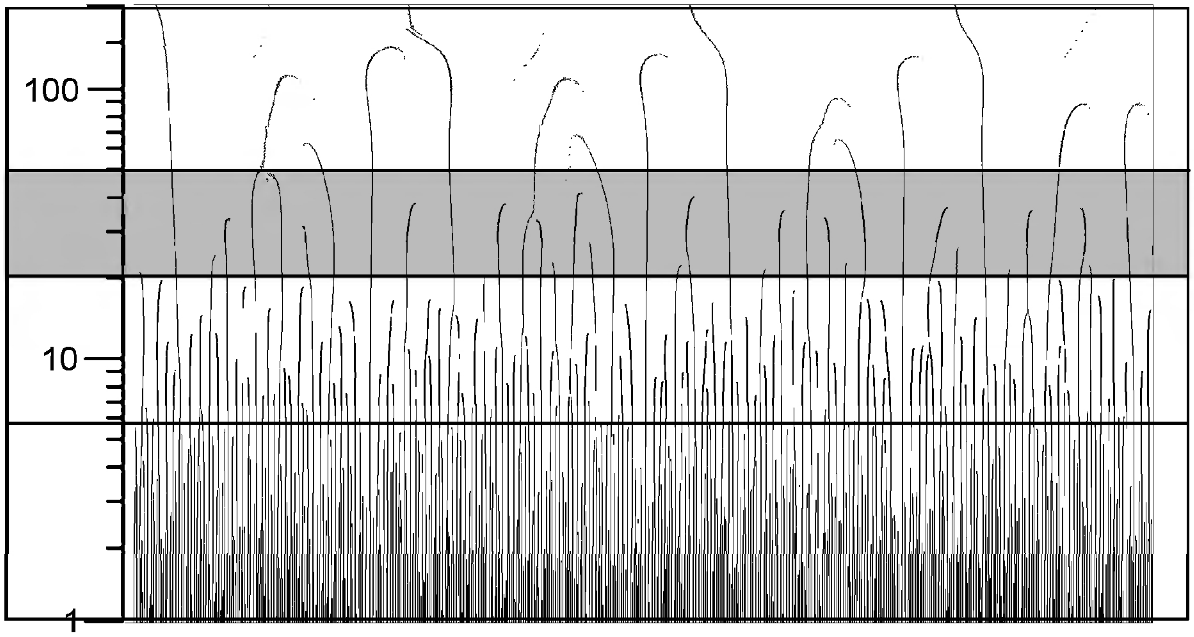

Fig. 3. *Extracting scales regions on the spectrum. Gray area contains wavelet image of main ECG pulses — p, q, R, s and T.*

6.2. Filtering

Dividing into layes can be efficiently exploited. First we transform analyzing singnal according with (1). Shift value *b* runs through all the values withing an area where the signal is defined. Scale parameter *a* must be varied in bounds according to the scales of features that we intent to extract from the signal.

This algorithm is called wavelet filtering, the programme perfoming it is wavelet filter. One more step is usually included into filtering mechanism. Before evaluating direct wavelet transfor we set to zero any wavelet coefficient $W_\psi(a,b)$ with absolute value less that some predefined cut-off level. Advanteges and shortcomings of VMWF based filters are described in [7].

Let us refer to images on Fig. 4 where results of filtering one period of ECG at different scales are shown. Gray curve is source data while bold one is the result.

Low scale filter (Fig. 4, a) corresponding bottom part of the spectrum select high frequency components only. It is non desired noise usually but in the case of ECG we see that the filter passes sharp QRS pulses as well.

Changes of scale it is possible to fully remove noises from the signal. At the same time we can corrupt useful signal (as it happened at Fig. 4, d).

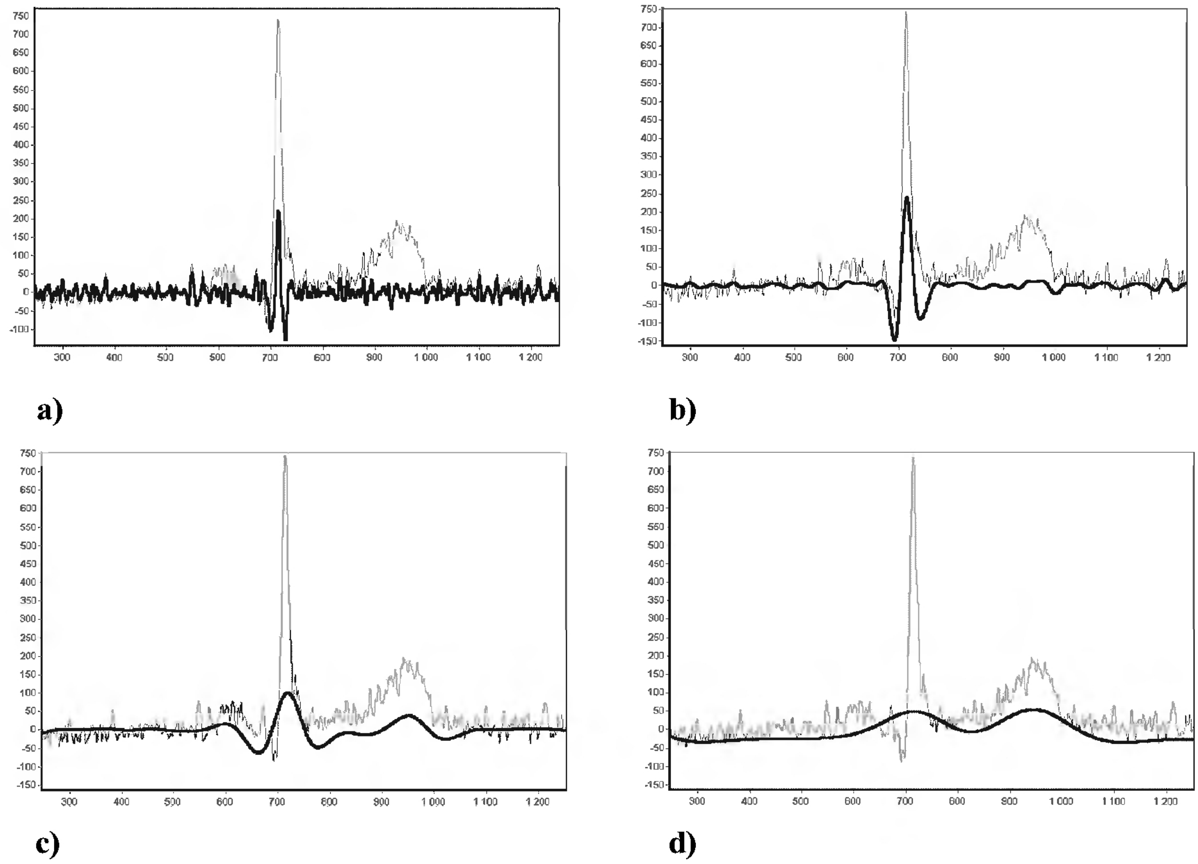

Fig. 4. *Selecting signal components*
a) at scales 1–7, b) 8–19, c) 22–45, d) 53–215.

To solve the problem of what scales to select it is easier to perform the direct wavelet transform at scales varying in wide regions. Then viewing the spectrum we can pick out those areas that image features we desire to obtain. And finally we perform inverse transform to recover the signal. Note that it is possible to take more than one stripe on the skeleton.

6.2.1. Removing curved baseline

Inverse wavelet transform (as it was mentioned in the section 2) reconstructs the signal without moments of low orders. In the case of VMWF we loose all the moments of the order less that the order of the wavelet according to (6).

This fearture of wavelets helps us to correct curved baseline of an ECG signal. These distortions comes from variations of resistance in electrical contact between sensor and human skin. Zero potential on the ECG usually coincides with an interval from the end of T pulse to the beginning of pulse p.

Fig. 5 shows how the base lines changes after passin the signal through wavelet filter. This simplifies measuring pulses especially in automated ECG analysis.

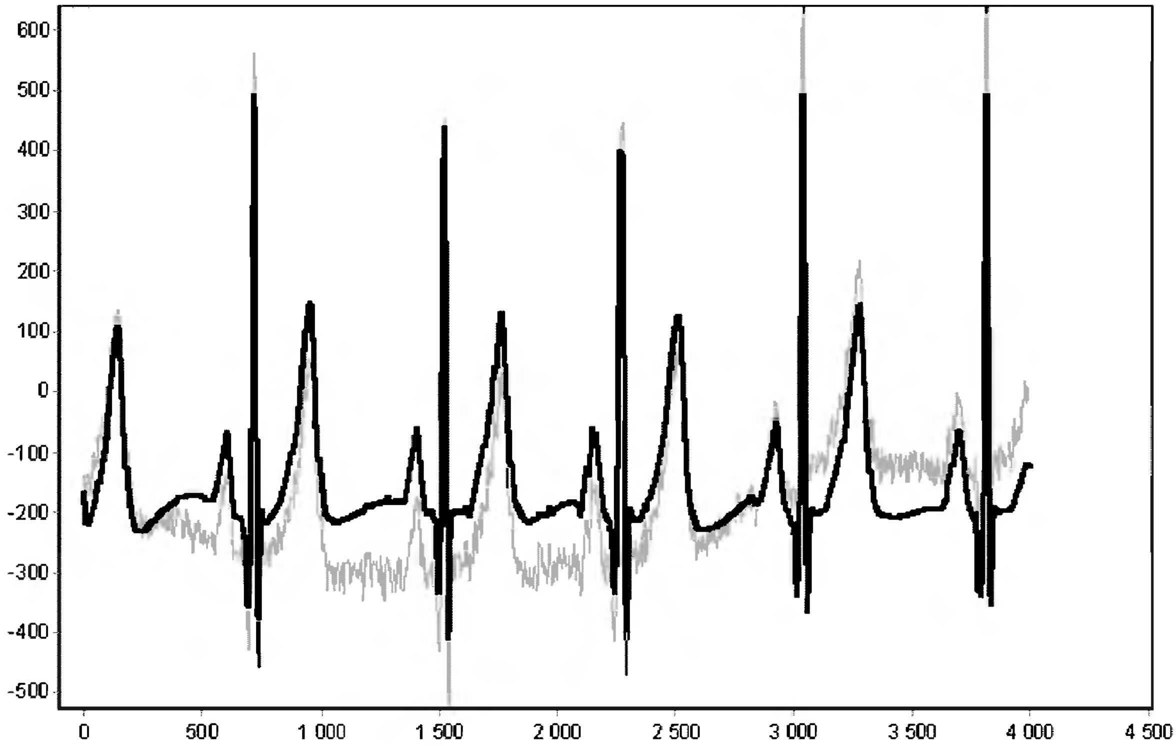

Fig. 5. *Recovering naseline of the ECG. Soucesignal (gray curve) is wavelet-filtered at scales 8 through 128.*

6.3. Applying wavelet filter to ECG data

Varying bounds of scale areas it is possible to tune the filter for specific problems, for ECG analysis at least three tasks might be solved:

1. **Denoising.** We need to filter the signal at scales larger than some level which is usually clear seen on the spectrum. Skeleton areas that correspond to noises are filled with short curves traces. Cut-off level is selected experementally.
2. **Recovering baseline.** This operation is the simpliews and does not require any additional manipulations with the filter. Baseline restoring is possible due to the features of VMWF wavelets.
3. **Extractingf *R–R* intervals.** Choosing scales for transformation is it possible to reach assured selection of QRS complect on the background of other parts on an ECG structure. Fig. 4, b shows that the position of R pulse agrees with maxima of the resultant signal.

7. Adaptive filtering

Primary wavelet filter described in the previous section takes into accound only frequency splitting of the signal, the way that Fourier transform does. It is enough if we e. g. reconstruct the baseline. Wavelet analysis allows us to consider localization of the features as well.

Refering back to Fig. 4 we can notice that while selecting noise (Fig. 4, a) the filter passes some part of the QRS complex. So if we try to remove the noise we loose data.

Top image on Fig. 6 shows how to split the data into several intervals according signal strength and positions of ECG pulses. Central area selects pulse R, middle one (280–520) surrounds QRS complex, and finally most wide interval includes the whole signal.

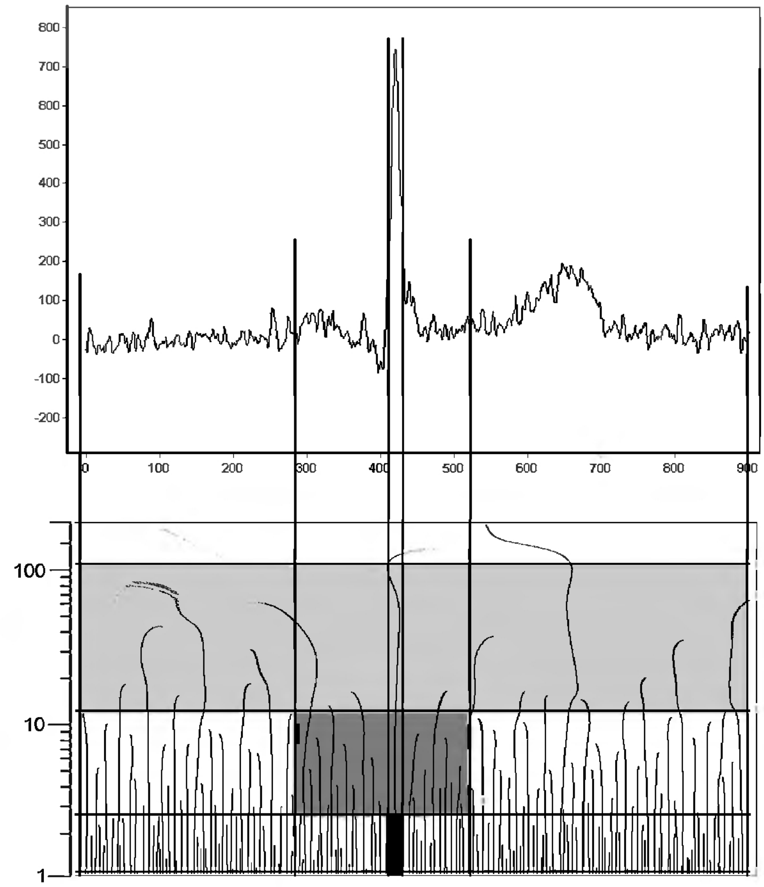

Fig. 6. *Marking areas of an ECG according with their meaning.*

Now we can select different cut-off levels in each area (see table 1). This allows the filter to safely pass QRS complex. Signal/noise ratio is higher than in other parts so we can select less cut-off level.

Table 1. *Scales and corresponding cut-off levels for adaptiev filter.*

Scale *a*	**Cut-off level**	**Area**
1,189–3,364	100	405–415
4–11,314	400	280–520
wider 13,454	250	0–900

One fragment of the recovered signal is presented on Fig. 7. On the left is the signal went from the filter with constant cut-off level within the bounds of the signal. On the right the same source data were passed through the filter set up as shown in Table 1. No doubt modification of filtering algorithms allows to reconstruct the signal much closer to the original.

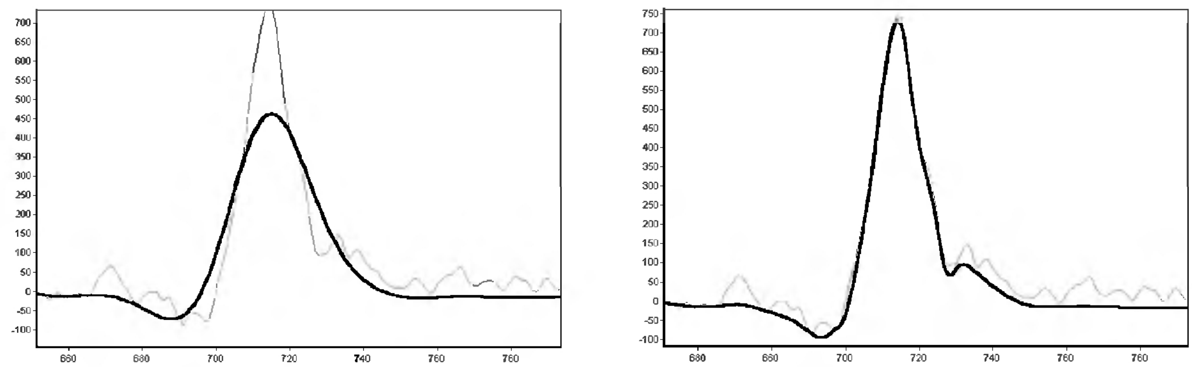

Fig. 7. *Restoring the signal with constant cut-off level (250) (on the left) and with adaptive filter (on the right).*

8. Classification of patients

Applying algorithms that were proposed above we have in mind that they are the initial part of ECG handling. Wavelet analysis anyhow provides methods that are able to make some conclusion without any additional procedures. Bright example of it is presented in [9].

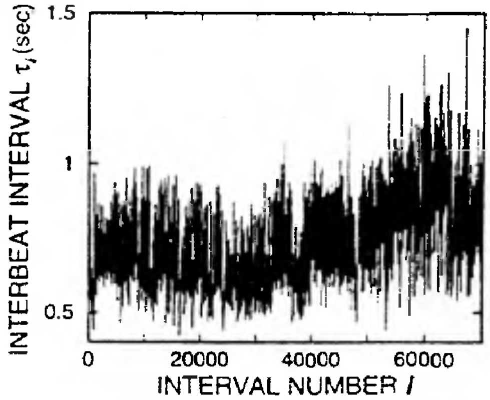

Fig. 8. *Record of R–R sequence.*

Sequences of *R–R* intervals (Fig. 8) registered within several hours were transformed into wavelet space. Then mean squeres were calculated at every scale m (discrete versions of wavelet transform were used):

$$\sigma_{\mathrm{wav}}(m)=\left[\frac{1}{1-N}\right]\sum_{n=0}^{N-1}\sqrt{W_{m,n}^{\mathrm{wav}}(s)-\left\langle W_{m,n}^{\mathrm{wav}}(s)\right\rangle^{2}}\,.$$

Putting these values onto the plot (see Fig. 9) showed that the distribution is weakly dependent on thetype of wevelets. Viewing the plot of *R–R* wavelet coefficient rms it is possible to say whether the patient suffers from any heart illness including predisposition to sudden death.

Distributions of wavelet coeffitients on Fig. 9 which was obtained from 27 records clearly separates lines in the area of scales 3 through 6. This fact allows to separate groups of patients.

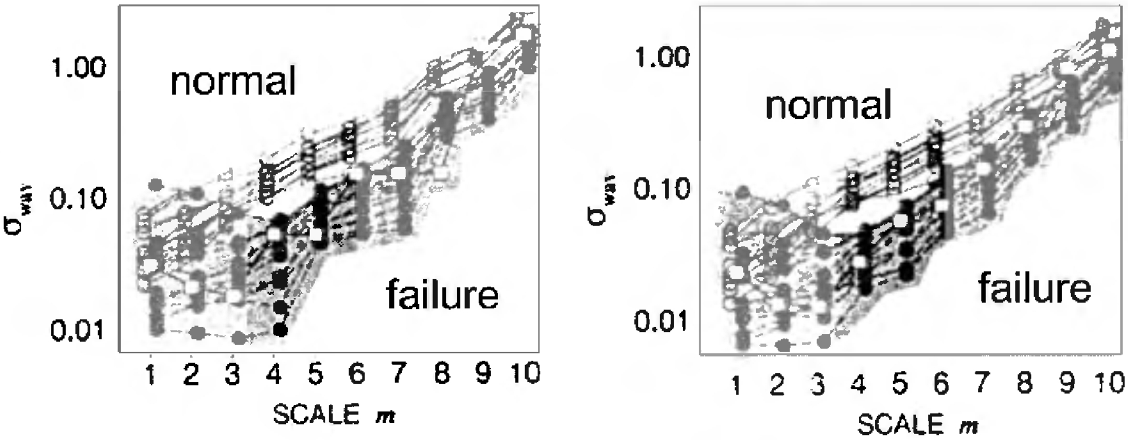

Fig. 9. *Rms of Wavelet coeffitients of different scales. On the left: Haar wavelet is used, on the right: Daubechies wavelet.*

9. References

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